

# Subdifferential Determination of a Primal Lower Regular Function on a Banach Space

Milen Ivanov<sup>1</sup>, Matey Konstantinov<sup>2</sup>, and Nadia Zlateva<sup>2</sup>

<sup>1</sup>Radian Life Technologies Ltd., Nicosia, Cyprus

<sup>2</sup>Faculty of Mathematics and Informatics, Sofia University, Sofia, Bulgaria

## Abstract

The primal lower regular functions are introduced by R. Poliquin in [2] under the name primal lower nice functions, for a modern account see [4, Chapter 11]. They are a useful abstraction of the framework for non-convex optimization proposed by R.T. Rockafellar, see e.g. [3], that is, the compositions of convex lower semicontinuous and twice smooth functions.

In [2] R. Poliquin shows that primal lower regular functions share many properties with the convex functions, including a local version of *Moreau-Rockafellar integrability theorem*. Namely, if the subdifferentials of two primal lower regular functions coincide in a neighbourhood of some point, then in a neighbourhood of that point the functions differ by a constant. It was proved in finite dimensions in [2] and later in [5] it was extended to Hilbert space setting, see also [4, p.1113]. These proofs, however, rely on the special properties of Moreau-Yosida regularization which allow transferring the subdifferential equality to the regularized functions, when the norm is Hilbertian. This does not work in a general Banach space.

We generalize to Banach space *Thibault-Zagrodny Theorem* that if  $f$  and  $g$  are primal lower regular functions and  $\partial f = \partial g$  locally, then  $f$  and  $g$  locally differ by a constant. The method we employ is based on *slopes*, see e.g. [1] and the references therein, and it works, because it does not require linear (or convex) structure.

## References

- [1] M. Ivanov, N. Zlateva: *Slopes and Moreau-Rockafellar theorem*, J. Convex Analysis 32/2 (2025) 359–374.
- [2] R. A. Poliquin: *Integration of subdifferentials of nonconvex functions*, Nonlinear Anal. Th. Meth. Appl. 17 (1991) 385–398
- [3] R. T. Rockafellar: *First- and second-order epi-differentiability in nonlinear programming*, Trans. Amer. Math. Soc. 307/1 (1988) 75–108.
- [4] L. Thibault, *Unilateral Variational Analysis in Banach Spaces. Part I: General Theory. Part II: Special Classes of Functions and Sets*, World Scientific (2023) ISBN: 978-981-125-816-9.

- [5] L. Thibault, D. Zagrodny: *Integration of subdifferentials of lower semicontinuous functions on Banach spaces*, J. Math. Analysis Appl. 189/1 (1995) 33–58.